

$$\frac{7+\sqrt{13}}{6} > \frac{7+3}{6} = \frac{10}{6} = \frac{5}{3} > 1.5$$

reject $\frac{7+\sqrt{13}}{6}$ because it is larger than 1.5

$$\frac{7-\sqrt{13}}{6} < \frac{7-3}{6} = \frac{4}{6} = \frac{2}{3} < 1.5$$

$$\frac{7-\sqrt{13}}{6} < \frac{6}{6} < 1 < 1.5 \quad \text{keep } \frac{7-\sqrt{13}}{6}$$

prove this is a max.

$$V''(x) = -28 + 24x$$

$$\text{Solve } V''(x) = 0$$

$$-28 + 24x = 0$$

$$24x = 28$$

$$x = \frac{7}{6}$$

check $x=1$

$$V''(x) < 0$$

so concave down for $x < \frac{7}{6}$.

$$\text{max volume is } V\left(\frac{7-\sqrt{13}}{6}\right) = \left(\frac{7-\sqrt{13}}{6}\right)\left(4 - 2\left(\frac{7-\sqrt{13}}{6}\right)\right)\left(3 - 2\left(\frac{7-\sqrt{13}}{6}\right)\right) \text{ in } \text{ft}^3$$